S.-T. Yau College Student Mathematics Contests 2018

Analysis and Differential Equations

Individual Overall

Please solve the following problems.

1. Let f be an analytic function in a neighborhood of $\bar{U}$ where $U = \{|z| < 1\}$. Show that if f is real on ∂U, then f must be constant.

2. Find the Green's function for

$$-u^{''} = f$$
$$u(0) = u(1), u^{'}(0) = u^{'}(1)$$
$$\int_0^1 u = 0, \int_0^1 f = 0$$

3. Let $B_R(0) = \{x \in \mathbb{R}^n : |x| < R\}$ for $n > 2$ and $R > 0$. Prove that there exists a constant C independent of R such that

$$\int_{B_R(0)} \frac{|v(x)|^2}{|x|^2} dx \leq C \int_{B_R(0)} (|v_r|^2 + R^{-2} v^2) dx$$

for any function $v \in C^\infty(B_R(0))$, where $v_r(x) = \frac{x \cdot \nabla v}{|x|}$.